INDIAN SCHOOL AL WADI AL KABIR
Assessment - 1 (2022 - 2023)

Class: XII   Sub: APPLIED MATHEMATICS (**241**)   Max Marks: 80
Date: 01.09.2022   **Practice Paper**   Time: 3 hr

General Instructions:
1. This question paper is divided in to 6 sections- A, B, C, D, E and F
2. Section A comprises of 16 objective type questions of 1 mark each.
3. Section B comprises of 8 MCQ type questions of 1 mark each.
4. Section C comprises of 8 questions of 2 marks each.
5. Section D comprises of 4 questions of 3 marks each.
6. Section E comprises of 4 questions of 5 marks each.
7. Section F comprises of 2 case study-based questions.
8. Internal choice has been provided for certain questions

SECTION – A
(Each Question Carries 1 Mark)

1   Priya can run 1 km in 3 min 10 sec and Preeti can cover the same distance in 3 min 20 sec. By what distance can Priya beat Preeti?

2   Now time is 6.00pm. What will be the time after 1476 hrs.

3   Find subtraction modulo 7 if a and b are 11 and 43 respectively

4   If A = $\begin{bmatrix} \frac{1}{3} & 2 \\ 0 & 2x-3 \end{bmatrix}$, B = $\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ and AB = $\mathbf{I_2}$, then find x

5   If $\begin{bmatrix} x+y+z \\ x+y \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$, then find the values of x, y and z.

6   A is a non-singular matrix of order 3 such that $A^2 = 3A$. $then\ the\ value\ of\ |A|\ is$ _____

7   If A = $\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ then find the determinant of the Matrix $A^2 - 2A$

8   Evaluate the integral $\int_0^1 \frac{x}{x^2+1} \, dx$

9   Find two positive numbers whose sum is 16 and whose product is as large as possible.

10   Find the given integral $\int \frac{x^3+5x^2-4}{x^2} \, dx$

11   The marginal revenue of a company is given by MR = $80 + 20x + 3x^2$, where x is the number of units sold for a period. Find the total revenue function R(x) if at x=2, R(x) = 240.

12   Ms. Manya deposited Rs.10,000 in a bank that pays 4% interest compounded continuous. How much amount will she get after 10 years? ($e^{0.4} = 1.4918$)

13   Degree of the given differential equations is $\left(\dfrac{dy}{dx}\right)^4 + 3y\left(\dfrac{d^2y}{dx^2}\right) = 0$

14   The rise in price before Diwali is an example for _____ trend

15   $(-6 \times 5) \pmod 7$ is _____

16   If A is a square matrix such that $A^2 = A$, then evaluate $(I + A)^2 - 3A$

SECTION – B
(Each MCQ Carries 1 Mark)

17   If $x \equiv 4 \pmod 7$, then the positive values of x are
a) $\{4,11,18, \ldots\}$ b) $\{11,18,25,\ldots\}$ c) $\{4,18,12, \ldots\}$ d) $\{1,8,15,\ldots\}$

18   The Matrix $A = \begin{bmatrix} 3 & -2 & 7 \\ -2 & 1 & -3 \\ 7 & -3 & 5 \end{bmatrix}$, is a
a) Symmetric Matrix   b) Skew-Symmetric matrix
c) Diagonal Matrix   d) Scalar Matrix

19   If $\begin{vmatrix} 3x & 4 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 4 & -3 \\ 5 & -2 \end{vmatrix}$, then x is
a)   -2   b) $\dfrac{4}{3}$ c)   3 or -3   d)   6 or -6

20   If A is a square matrix of order 3 such that $A.(\text{adj. } A) = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$,

then $|A|$ is equal to
a)   -27   b)   9   c)   -3   d)   3

21   Insert the appropriate sign of inequality: $\sqrt{3}(\sqrt{50} + \sqrt{32})$ _____ $3\sqrt{54} + 2\sqrt{24}$
a) $<$ b) $>$ c) $\leq$ d) $\geq$

22   Evaluate the integral $\int_0^1 x.e^x \, dx$
a) $x.e^x$ b) e^x c)   1   d)   0

23   Solution of the differential equation: $y \log y \, dx - x \, dy = 0$
a) $y = e^{cx}$ b) $x = e^{cy}$ c) $y = e^x + C$ d)   None of These

24 A fire in a factory delaying production for some time is
    a) Long term trend   b) Cyclical trend
    c) Seasonal Trend   d) Irregular trend

SECTION – C
(Each Question Carries 2 Marks)

25 Find the remainder when (127 x 137 x 23 x 50 x 235 x 15) is divided by 7.

- OR –

Find the value of 612 (mod) 7

26 A boy is blowing air into a spherical balloon and thus the radius r of the balloon is changing, then find the rate of change of surface area of the balloon with respect to the radius r, when r = 5cm

27 If matrix A = $\begin{bmatrix} 6 & x & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{bmatrix}$ is a singular matrix, then find the value of x

- OR –

Find X and Y if X + Y = $\begin{bmatrix} -1 & 13 \\ 2 & 4 \end{bmatrix}$ and X + Y = $\begin{bmatrix} 5 & -8 \\ 3 & 0 \end{bmatrix}$

28 Find the consumers' surplus for the demand function $p = 25 - x - x^2$ when $p_0 = 19$.

29 The amount of radiocarbon present after t years is given by A = $A_0\, e^{\left(\frac{-t \log 2}{5700}\right)}$ where A_0 is the amount present in the living plants and animals. Find the half-life of radiocarbon.

30 If A = $\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$

- OR –

Express matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ -4 & -1 & 0 \\ 3 & 5 & 1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix

31 For the values 15, 24, 18, 33, 42 write the three years moving averages.

32 An Ice cream manufacturing firm assesses its variable cost to be 'x' times the sum of 30 and 'x', where 'x' is the number of Ice creams produced, also the cost incurred on storage is Rs.2000. Find the total cost function and the marginal cost when 50 Ice creams are produced

SECTION – D
(Each Question Carries 3 Marks)

33 A boat covers 32 km upstream and 36 km downstream in 7 hours. Also it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

34 A container contains 40 *litre* milk. From this container 4 *litre* milk was taken out and replaced with water. This process was repeated further two more times. How much milk is there in the container now?

- OR –

The average salary per head of the entire staff of a small factory including the supervisor and labours is Rs 5750. The average salary per head of the supervisor is Rs 20,000 and that of the labours is Rs 5000. Find the number of labours in the factory if there are 4 supervisors.

35 A cistern can be filled by two pipes *A* and *B* in 12 minutes and 15 minutes respectively. Another tap *C* can empty the full tank in 20 minutes. If the tap *C* is opened 5 minutes after the pipes *A* and *B* are opened, find when the cistern becomes full?

- OR –

In a 900 metres race, *A* gives *B* a start of 150 metres and defeats him by 50 seconds. If the speed of *A* is 4.5m/sec then find the speed of *B*.

36 Calculate the 3-year moving averages for the loans (In lakh Rs) issued by co-operative banks for farmers in different states of India based on the values given below.

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014
Loan Amount in Lakhs	41.85	40.2	38.12	26.5	55.5	23.6	28.36	33.31	41.1

SECTION – E
(Each Question Carries 5 Marks)

37 A school plans to award Rs 6000 in total to its students to reward for certain values - honesty, regularity and hard work. When three times the award money for hard work is added to the award money given for honesty amounts to Rs 11000. The award money for honesty and hard work together is double the award money for regularity. Find the prize money for each category of award?

- OR –

Given below are the consumer price index numbers (CPI) of the industrial workers.

Year	2014	2015	2016	2017	2018	2019	2020
Index Number	145	140	150	190	200	220	230

38 A wire 40 m length is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the square and the circle is minimum? (use $\pi = 3.14$)

- OR –

Find the intervals in which the function $f(x) = \frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x$ is increasing and decreasing.

39 Evaluate the following Integrals

a) $\int_0^1 \frac{\log x}{\log x + \log(1-x)} \, dx$

b) $\int \frac{(x-1)(x-2)}{(x-3)(x-4)} \, dx$

40 In a certain culture of bacteria the rate of increase is proportional to the number present. It is found that there are 10,000 bacteria at the end of 3 hours and 40,000 bacteria at the end of 5 hours. How many bacteria were present in the beginning?

SECTION – F
(CASE STUDY - Each Question Carries 4 Marks)

41 Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50m, then its area will remain same, but if length is decreased by 10m and breadth is decreased by 20m, then its area will decrease by 5300 m². Based on the information given above, answer the following questions

1). The equations in terms of X and Y are
a. x − y = 50, 2x − y = 550   b. x − y = 50, 2x + y = 550
c. x + y = 50, 2x + y = 550   d. x + y = 50, 2x + y = 550

2) Which of the following matrix equation is represented by the given information

a. $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -50 \\ -550 \end{bmatrix}$

3). The value of x (length of rectangular field) is
a. 150m   b. 400m   c. 200m   d. 320m

4). The value of y (breadth of rectangular field) is
a. 150m.   b. 200m.   c. 430m.   d. 350m

42 Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 11.30 pm which was 94.6⁰F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4⁰F. The room in which the cat was put is always at 70⁰F. The normal temperature of the cat is taken as 98.6⁰F when it was alive. The doctor estimated the time of death using *Newton law of cooling which is governed by the differential equation:* $\frac{dT}{dt} \propto (T - 70)$, where 70⁰F is the room temperature and T is the temperature *of the object* at time *t.*

Substituting the two different observations of T and t made, in the solution of the differential equation $\frac{dT}{dt} = k(T - 70)$ where k is a constant of proportion, time of death is calculated.

1) State the degree of the above given differential equation

a. 0   b. 1   c. 2   d. 3

2) If the temperature was measured 2 hours after 11.30pm, will the time of death change?

a. Yes   b. No   c. Not measurable

3) The solution of the differential equation $\frac{dT}{dt} = k(T - 70)$ is given by,

a. $\log | T - 70| = kt + C$ b. $\log | T - 70| = \log |kt| + C$

c. $T - 70 = kt + C$ d. $T - 70 = kt\ C$

4) If t = 0 when T is 72, then the value of c is

a. -2   b. 0   c. 2   d. log 2